Is the $U_A(1)$ symmetry restored at finite temperature or density?

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Abstract. We investigate the full U(3)$\otimes$U(3) chiral symmetry restoration, at finite temperature and density, on the basis of the three flavor Nambu-Jona-Lasinio model with the anomaly term given by the 't Hooft interaction. We implement a temperature (density) dependence of the anomaly coefficient motivated by lattice results for the topological susceptibility. The results suggest that the axial part of the symmetry is restored before the possible restoration of the full U(3)$\otimes$U(3) chiral symmetry can occur.

The important role of Quantum Chromodynamics (QCD) at finite temperature and density to describe relevant features of particle physics in the early universe, in neutron stars and in heavy-ion collisions, is nowadays more and more recognized, bringing together researches in lattice QCD, effective models like the Nambu-Jona-Lasinio (NJL) model, and compact star physics calculations. In fact, restoration of symmetries and deconfinement are expected to occur under extreme conditions that may be achieved in ultra relativistic heavy-ion collisions or in the interior of neutron stars. An interesting question is whether both chiral SU$(N_f)\otimes$SU(N_f) and axial $U_A(1)$ symmetries are restored and which observables could carry information about the possible restorations.

Several studies have been done linking the decrease with temperature of the topological susceptibility, χ, with the restoration of the $U_A(1)$ symmetry [1]. There are also preliminary lattice results which indicate the existence of a drop in the behavior of χ with increasing baryonic density [2].

We perform our calculations in the framework of an extended SU(3) Nambu–Jona-Lasinio model Lagrangian density that includes the 't Hooft determinant:

$$\begin{aligned}\mathscr{L} &= \bar{q}(i\gamma.\partial-\hat{m})q+\frac{g_S}{2}\sum_{a=0}^{8}[(\bar{q}\lambda^a q)^2+(\bar{q}i\gamma_5\lambda^a q)^2] \\ &+ g_D\{\det[\bar{q}(1+\gamma_5)q]+\det[\bar{q}(1-\gamma_5)q]\}. \qquad (1)\end{aligned}$$

By using a standard hadronization procedure, an effective meson action is obtained, leading to gap equations for the constituent quark masses and to meson propagators from which several observables are calculated [3].

In the present work we follow the methodology of [4, 5], and extract the temperature dependence of the anomaly coeficient g_D from the lattice results for the topological susceptibility [1]. Other dependences for g_D were studied in [6]. At temperatures around $T\approx 200$ MeV the mass of the light quarks drops to the current quark mass, indicating a

washed-out crossover. The strange quark mass also starts to decrease significantly in this temperature range, however even at $T = 400$ MeV it is still 2 times the strange current quark mass. So, chiral symmetry shows a slow tendency to get restored in the s sector. In fact, as $m_u = m_d < m_s$, the (sub)group SU(2)$\otimes$SU(2) is a much better symmetry of the NJL Lagrangian. So, the effective restoration of the above symmetry implies the degeneracy between the chiral partners (π^0, σ) and (a_0, η) which occurs around $T \approx 250$ MeV. At $T \approx 350$ MeV both a_0 and σ mesons become degenerate with the π^0 and η mesons, showing an effective restoration of both chiral and axial symmetries. So, we recover the SU(3) chiral partners (π^0, a_0) and (η, σ) which are now all degenerated. However, the η' and f_0 masses do not yet show a clear tendency to converge in the region of temperatures studied [7].

Recent calculations on lattice QCD at finite chemical potential motivates also the study of the restoration of the U$_A$(1) symmetry at finite density. Since there are no firmly lattice results for the density dependence of χ, to be used as input, we have to extrapolate from our previous results for the finite temperature case and proceed by analogy. Here we present an example (see Fig. 1, left panel) where we consider quark matter simulating "neutron" matter. This "neutron" matter is in β–equilibrium with charge neutrality, and undergoes a first order phase transition [3]. To begin with, we calculate the mixing angles for scalar and pseudoscalar mesons, θ_S and θ_P, respectively. We observe that θ_S starts at 16° and increases up to the ideal mixing angle 35.264°. A different behavior is found for the angle θ_P that changes sign at $\rho_B \approx 4\rho_0$. In fact, it starts at -5.8° and goes to the ideal mixing angle 35.264°, leading to a change of identity between η and η'. We think this result might be a useful contribution for the understanding of the somewhat controversial question: under extreme conditions will the pion degenerate with η or η'? We found that the change of sign and the corresponding change of identity between η and η', effects that we do not observe in the finite temperature case, is related to the small fraction of the strange quarks that only appear in the medium for $\rho_B \approx 4\rho_0$ [3].

The meson masses, as function of the density, are plotted in Fig. 1, right panel. The results for constant g_D are also presented (middle panel) for comparison purposes. The SU(2) chiral partners (π^0, σ) are bound states and become degenerated at $\rho_B = 3\rho_0$. With respect to the SU(2) chiral partners (η, a_0), the a_0 meson is always a purely non strange quark system. For $\rho_B < 0.8\rho_0$ a_0 is above the continuum and, when $\rho_B \geq 0.8\rho_0$, a_0 becomes a bound state. At $\rho_B = 0$, the η has a strange component and, as the density increases, η becomes degenerated with a_0 at $4.0\rho_0 \leq \rho_B \leq 4.8\rho_0$ as expected. In this range of densities (η, a_0) and (π^0, σ) are all degenerated. Suddenly the η mass separates from the others becoming a purely strange state. This is due to the behavior of θ_P that changes the sign and goes to 35.264° at $\rho_B \approx 4.8\rho_0$. On the other hand, the η', that starts as an unbounded state and becomes bounded at $\rho_B > 3.0\rho_0$, turns into a purely light quark system and degenerates with π^0, σ and a_0 mesons. Taking into account the presented arguments, we conclude that the U$_A$(1) symmetry is effectively restored at $\rho_B > 4\rho_0$ [7]. In fact, the U$_A$(1) violating quantities show a tendency to vanish, which means that the four meson masses are degenerated and the topological susceptibility goes to zero. Without the restoration of the axial symmetry, the a_0 (σ) mass was moved upwards and never met the π^0 (η') mass as can be seen in Fig. 1, middle panel. We remember that the determinant term acts in an opposite way for the scalar and pseudoscalar mesons.

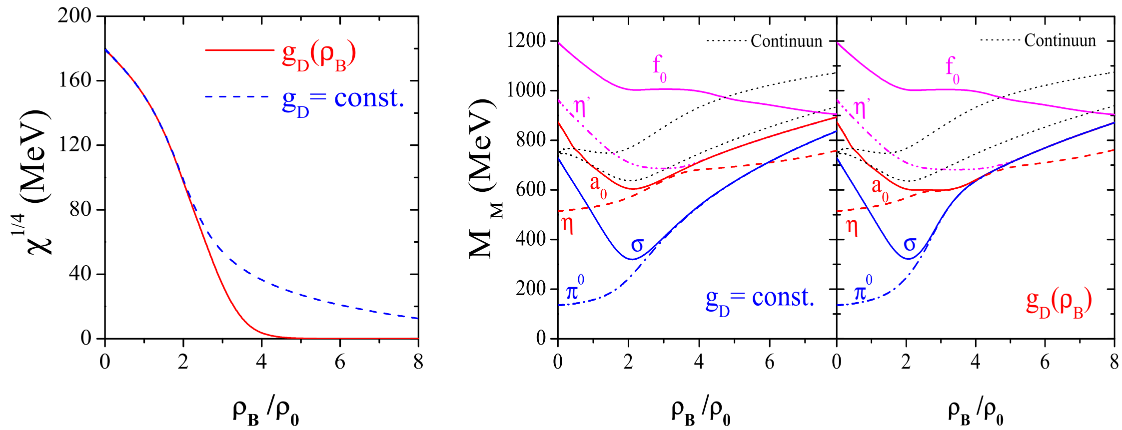

FIGURE 1. Topological susceptibility (left panel): the solid (dashed) line represents our fitting with constant (density dependent) g_D. Meson masses, as functions of density, with g_D constant (middle panel) and $g_D(\rho_B)$ (rigth panel). The dotted lines indicate the density dependence of the limits of the Dirac sea continua, defining $q\bar{q}$ thresholds for a_0 and η' mesons.

In summary, we have implemented a criterion which combines a lattice-inspired behavior of the topological susceptibility with the convergence of appropriate chiral partners to explore effective restoration of symmetries. However, the role of $U_A(1)$ symmetry for finite temperature, and mainly for finite density media, has not been so far investigated and this question is still controversial and not settled yet. We hope that new studies, especially lattice based and experimental ones, can finally clarify it.

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